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Study of the critical properties of the Quantum Hall Fluid in the framework of a dual statistical model. [1]

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Abstract

By using Renormalization Group methods we analyze the outcome of a recent proposal of describing the Quantum Hall fluid in terms of a dual plasma which embodies dyons as effective degrees of freedom. As a consequence the physical interpretation of the two parameters of the model as the longitudinal and the Hall conductances is made clear. Parameters' scaling roperties allow for the determination of the critical index for the localization length after a mapping of our statistical model onto a classical percolation model. The universality of the critical properties, supported by recent experiments, is a consequence of the infinite discrete symmetry of the model $SL(2,Z)$ (generalized duality), as noticed in a previous paper.

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1 Introduction.

Several years ago it was discovered that the ground-state wave function proposed by Laughlin [1] for a Hall system at filling $f = 1/m$, where m is an odd integer, is simply described (at least for its analytic part) in terms of correlators of primary fields of a two-dimensional Conformal Field Theory (**2D CFT**) with central charge $c = 1$ [2, 3].

Within such a framework a more straightforward and physical picture of the Laughlin state was given in terms of effective degrees of freedom, which play the role of "collective modes" of the otherwise strongly correlated electrons system. Laughlin's physical idea of associating a magnetic flux to the electron [1] is in agreement with a detailed analysis of the ground state properties of the Hall system with doubly periodic boundary conditions [4]. More specifically for filling $f = 1/m$ the allowed magnetic translations have a finite step which defines an "elementary cell" for the electron with a linked magnetic flux just equal to $m\Phi_0$, where the "elementary flux" Φ_0 is expressed in terms of universal quantities as $\Phi_0 = \frac{hc}{e}$. The previous observation suggests a picture of the Hall fluid at a given filling in terms of "dyons" (i.e., objects carrying both electric and magnetic charge) as the relevant degrees of freedom. Then the conduction properties are simply reproduced by employing the transformation properties of the ground state under the finite magnetic translations and its topological properties are easily derived.

Furthermore the phenomenological "laws of corresponding states" [5] suggest an unified description of the Hall fluid at integer as well as at fractional filling in terms of a discrete symmetry. Such a symmetry has been used by several authors [5, 6] but only on a qualitative ground.

In this paper we analyze the consequences of a recent proposal of describing the Quantum Hall fluid which incorporates in a simple way both the phenomenon of dyon condensation and the discrete symmetry $SL(2, Z)$ [7]. By employing simple Renormalization Group (**RG**) techniques we are able to study the flow of the parameters appearing in the model. As a result we are let to identify the RG Infrared (**IR**) attractive fixed points as stable condensates of the Hall fluid at the plateaux. The $SL(2, Z)$ symmetry acts as in [8, 9], mapping the IR fixed points into one another (a simple connection between the duality transformations of the model and the laws of corresponding states has been given in [7]). Furthermore we study the properties of the model around the RG repulsive fixed points, once identified as transition points between plateaux. After mapping our model onto a classical percolation model, the scaling properties of the longitudinal and Hall conductances (corresponding to the two parameters appearing in the model) allow us to fix the critical index for the localization length at the value $\nu = \frac{4}{3}$. Universality of the result again is assured by the discrete $SL(2, Z)$ symmetry, mapping repulsive fixed points into one another.

For clarity sake we should make an obvious comment: our two-dimensional model is suitable for a description of the equilibrium properties of the Hall system.

The paper is organized as follows:

- In section 2 we review the construction of a non-trivial formulation of the Cardy-Rabinovici (**CR**) model where we split the field configuration in a uniform "background" part plus a fluctuating field in order to describe a Coulomb Gas of charges of the same sign neutralized by a classical background of opposite sign [7]. We observe the existence of an infinite discrete symmetry $SL(2,Z)$.

- Section 3 contains a detailed RG analysis of the model which allows for a simple description of its long distance properties and for a physical interpretation of the parameters appearing in the model. As outcome we find that there are stable phases corresponding to IR RG fixed points. The $SL(2,Z)$ symmetry maps those non-trivial fixed points into one another suggesting a unified picture of them in terms of a 2D CFT with central charge $c=1$.
- In section 4 our model, taken at the transition point, is mapped onto a percolation model and the critical index relative to the scaling of the localization length is derived.
- In section 5 we give a summary of the results and address some open questions.
- In the Appendix some properties of the $SL(2,Z)$ fixed points are discussed.

2 Construction and properties of the dual plasma model.

2.1 Cardy-Rabinovici model.

Let us first remind how Cardy and Rabinovici build their 2D model starting from a $U(1)$ gauge theory with both electric and magnetic matter coupled by a θ term [8, 9]. Then we will search for a non trivial extension of such a model in which a fixed background is generated.

The explicit Euclidean action is given by:

$$S[A^j,\vec{S}^j,n^j]\equiv\frac{1}{2g}\int d^2r\sum_{j=3,4}(\partial_\beta A^j+S^j_\beta)^2-i\int d^2r\sum_{j=3,4}n^jA^j-i\frac{\theta}{2\pi}\int d^2r\epsilon^{ij}\epsilon_{\beta\gamma}S^i_\beta\partial_\gamma A^j \quad , \tag{1}$$

$$(j=3,4)$$

where ϵ is the antisymmetric tensor in 2D, A^j are a pair of scalar fields and $S^j_\beta(\vec{r})$ are the "magnetic frustration fields" satisfying the constraints:

$$\epsilon_{\beta\gamma}\partial_\beta S^3_\gamma(\vec{r})-m^4(\vec{r})=0$$

$$\epsilon_{\beta\gamma}\partial_\beta S^4_\gamma(\vec{r})+m^3(\vec{r})=0 \tag{2}$$

while $n^j(\vec{r})$ and $m^j(\vec{r})$ are the electric and magnetic charge densities.

The densities (n^j, m^j) are constrained by the neutrality condition (required to make the system IR stable):

$$\int d^2r\ n^j(\vec{r}) = \int d^2r\ m^j(\vec{r}) = 0 \quad . \tag{3}$$

The Coulomb gas representation, where the role of the electric and magnetic charges is emphasized, is defined by:

$$e^{-S_{CG}(n^j,m^j)} = \int \prod_{j=3,4} \mathcal{D}A^j \int \prod_{\alpha=1,2} \mathcal{D}S^j_\alpha \delta(\epsilon_{\beta\gamma}\partial_\beta S^j_\gamma - \epsilon^{ij}m^i) e^{-S[A^j,\vec{S}^j,n^j]} \quad . \tag{4}$$

It is straightforward to evaluate the path integrals in eq.(4) and obtain:

$$S_{CG}[n^j,m^j] = \frac{g}{2}\int d^2rd^2r' \sum_{j=3,4} (n^j(\vec{r}) + \frac{\theta}{2\pi}m^j(\vec{r}))(n^j(\vec{r}\,') + \frac{\theta}{2\pi}m^j(\vec{r}\,'))G(\vec{r}-\vec{r}\,') + \tag{5}$$
$$\frac{1}{2g}\int d^2rd^2r' \sum_{j=3,4} m^j(\vec{r})m^j(\vec{r}\,')G(\vec{r}-\vec{r}\,') + i\int d^2rd^2r'\epsilon^{ij}n^i(\vec{r})m^j(\vec{r}\,')\varphi(\vec{r}-\vec{r}\,') \quad ,$$

where $G(\vec{r})$ and $\varphi(\vec{r})$ are the "longitudinal" and "transverse" Green-Feynman functions in 2D given by:

$$G(\vec{r}) = \ln\left(\frac{|\vec{r}|}{a}\right) \quad , \ \varphi(\vec{r}) = \arctan\left(\frac{y}{x}\right) \quad , \tag{6}$$

here a is a cutoff. The last term in eq.(6) is the (imaginary) Bohm-Aharonov term. Also notice that for $\theta = 0$ the standard Coulomb gas for both electric and magnetic charges is reproduced (see [10]).

2.2 The model with harmonic background.

To obtain a magnetic charge distribution consistent with Laughlin's description of the Hall fluid, i.e. the one-component plasma where all the vortices have the same magnetic charges and interact with an uniform external background of opposite sign which exactly neutralizes their total magnetic charge [1], we have to modify the CR model.

As a first step we introduce the background in the model described by the action of eq.(1) which, for $\theta = 0$, is given by:

$$S = \frac{1}{2g}\sum_{j=3,4}\int d^2r(\vec{\nabla}A^j + \vec{S}^j)^2 - i\int d^2rn^jA^j \tag{7}$$

where $\vec{S}^j$ must obey the constraint given by eq.(2).

The next step is the splitting of the charge densities into an uniform and a variable term:

$$m^j(\vec{r}) = \bar{m}^j + \mu^j(\vec{r}) \quad n^j(\vec{r}) = \bar{n}^j + \nu^j(\vec{r}) \tag{8}$$

and the neutrality condition of eq.(3) can be now rewritten as:

$$\int d^2 r \mu^j(\vec{r}) + \bar{M}_j = 0 \quad ; \quad \int d^2 r \nu^j(\vec{r}) + \bar{N}_j = 0 \tag{9}$$

where $\bar{M}^j = A\bar{m}^j$ and $\bar{N}^j = A\bar{n}^j$, A being the area of the sample and integration over the whole sample is understood.

Also by defining:

$$\vec{S}^j(\vec{r}) = \vec{\bar{S}}^j(\vec{r}) + \vec{\sigma}^j(\vec{r})$$

where the uniform part $\bar{S}^j$ obeys the equation

$$\epsilon_{\beta\gamma}\partial_\beta \bar{S}^j_\gamma(\vec{r}) = \epsilon^{jk}\bar{m}^k$$

and $\sigma(\vec{r})$ is the fluctuating part, we obtain:

$$\frac{1}{2g}\sum_{j=3,4}\int d^2 r(\vec{\nabla}A^j + \vec{S}^j)^2 =$$

$$\frac{1}{2g}\sum_{j=3,4}\int d^2 r(\vec{\nabla}A^j + \vec{\sigma}^j)^2 + \frac{1}{g}\int d^2 r \vec{\bar{S}}^j \cdot (\vec{\nabla}A^j + \vec{\sigma}^j) + \frac{1}{2g}\int d^2 r(\vec{\bar{S}}^j)^2 \tag{10}$$

The integral $\frac{1}{2g}\int d^2 r(\vec{\bar{S}}^j)^2$ is a simple number independent of the field configuration we shall disregard it. Furthermore the term $\frac{1}{g}\int d^2 r \vec{\bar{S}}^j \cdot (\vec{\nabla}A^j)$ will be zero once we fixed $\vec{\bar{S}}^j$ in such a way that $\vec{\nabla}\cdot\vec{\bar{S}}^j = 0$.

The term $\frac{1}{g}\int d^2 r \vec{\bar{S}}^j \cdot \vec{\sigma}^j$ gives instead:

$$-\sum_{j=3,4}\frac{\bar{m}^j}{4g}\int d^2 r \mu^j(\vec{r}) r^2$$

In order to work out the term:

$$\tilde{S} = \frac{1}{2g}\sum_{j=3,4}\int d^2 r(\vec{\nabla}A^j + \vec{\sigma}^j)^2 - i\sum_{j=3,4}\int d^2 r(\bar{n}^j + \nu^j)A^j$$

we define $A^j = a^j + \bar{A}^j$, where $\bar{A}^j$ is the background field satisfying:

$$\nabla^2 \bar{A}^j + ig\bar{n}^j = 0 \tag{11}$$

After imposing isotropic boundary conditions for $\bar{A}^j$ we get:

$$\bar{A}^j = -ig\frac{\bar{n}^j}{4}r^2 \tag{12}$$

Then the action in the Coulomb Gas representation can be written as follows:

$$S_{CG} = -\sum_{j=3,4}\frac{1}{2}\int d^2r\int d^2r'\left[\frac{\mu^j(\vec{r})\mu^j(\vec{r}')}{g}+g\nu^j(\vec{r})\nu^j(\vec{r}')\right]G(\vec{r}-\vec{r}')-$$

$$i\epsilon^{ij}\int d^2r\int d^2r'\mu^i(\vec{r})\nu^j(\vec{r}')\varphi(\vec{r}-\vec{r}')-\sum_{j=3,4}\frac{\bar{m}^j}{4g}\int d^2r\mu^j(\vec{r})r^2-$$

$$\frac{g\bar{n}^j}{4}\int d^2r\nu^j(\vec{r})r^2 \tag{13}$$

Defining for the variable part of the charge densities:

$$\mu^j(\vec{r})=\sum_{i=1}^N\mu_i^j\delta(\vec{r}-\vec{r}_i)\quad \nu^j(\vec{r})=\sum_{i=1}^N\nu_i^j\delta(\vec{r}-\vec{r}_i) \tag{14}$$

we finally get for the partition function of N particles in the presence of a background:

$$Z_N=\int\prod_{k=1}^N\frac{d^2r_k}{a^2}\exp\{\frac{1}{2}\sum_{j=3,4}\sum_{i\neq k=1}^N\left[\left(\frac{\mu_i^j\mu_k^j}{g}+g\nu_i^j\nu_k^j\right)\ln\left|\frac{\vec{r}_i-\vec{r}_k}{a}\right|\right]$$

$$+i\epsilon^{rj}\sum_{i\neq k=1}^N\mu_i^r\nu_k^j\varphi(\vec{r}_i-\vec{r}_k)\}\times$$

$$\exp\left\{\frac{1}{4}\sum_{j=3,4}\left[\frac{\bar{m}^j}{g}\sum_{k1}^N\mu_k^jr_k^2+g\bar{n}^j\sum_{k=1}^N\nu_k^jr_k^2\right]\right\} \tag{15}$$

The presence of a θ-term introduces an interaction between electric and magnetic charges obtained by the usual substitution:

$$\nu_i^j\rightarrow\nu_i^j+\frac{\theta}{2\pi}\mu_i^j$$

Therefore the complete form for the partition function for N particles is given by:

$$Z_N=\int\prod_{k=1}^N\frac{d^2r_k}{a^2}\exp\{\frac{1}{2}\sum_{j=3,4}\sum_{i\neq k=1}^N\left[\left(\frac{\mu_i^j\mu_k^j}{g}+g(\nu_i^j+\frac{\theta}{2\pi}\mu_i^j)(\nu_k^j+\frac{\theta}{2\pi}\mu_k^j)\right)\ln\left|\frac{\vec{r}_i-\vec{r}_k}{a}\right|\right]$$

$$+i\epsilon^{rj}\sum_{i\neq k=1}^N\mu_i^r(\nu_k^j+\frac{\theta}{2\pi}\mu_k^j)\varphi(\vec{r}_i-\vec{r}_k)\}\times$$

$$\exp\left\{\frac{1}{4}\sum_{j=3,4}\left[\frac{\bar{m}^j}{g}\sum_{k=1}^{N}\mu_k^j r_k^2 + g(\bar{n}^j + \frac{\theta}{2\pi}\bar{m}^j)\sum_{k=1}^{N}(\nu_k^j + \frac{\theta}{2\pi}\mu_k^j)r_k^2\right]\right\} \tag{16}$$

and the neutrality condition given by eq.(9), with the help of eq.(14) can be written as:

$$\sum_{i=1}^{N}\mu_i^j + \bar{M}^j = 0 \quad ; \quad \sum_{i=1}^{N}\nu_i^j + \bar{N}^j = 0 \tag{17}$$

In eq.(16) we notice the presence of a harmonic background term which has the correct form in order to describe the Hall fluid at the plateaux [1]. Details can be found in [7].

2.3 Discrete symmetries of the model: the modular group $SL(2,Z)$.

We now study the discrete symmetries of our model. It has been already noticed that the introduction of the θ angle allows for an extension of the usual electric-magnetic duality to a generalized discrete non-abelian symmetry which we shall refer to as generalized duality and which is described by the modular group $SL(2,Z)$ [8, 9] that acts on the complete action of the model, eq.(16).

In order to study the symmetries of the model we have to look at the explicit form of the action. We then easily find the following symmetry transformations:

- Periodicity $\hat{T}$:

$$\frac{\theta}{2\pi} \to \frac{\theta}{2\pi} + 1 \quad ; \quad \frac{1}{g} \to \frac{1}{g}$$

$$\bar{n}^j \to \bar{n}^j - \bar{m}^j \quad ; \quad \bar{m}^j \to \bar{m}^j$$

$$\{\nu^j\} \to \{\nu^j\} - \{\mu^j\} \quad ; \quad \{\mu^j\} \to \{\mu^j\} \tag{18}$$

- Duality $\hat{S}$:

$$\frac{1}{g} \to \frac{\frac{1}{g}}{\left(\frac{1}{g}\right)^2 + \left(\frac{\theta}{2\pi}\right)^2} \quad ; \quad \frac{\theta}{2\pi} \to \frac{-\frac{\theta}{2\pi}}{\left(\frac{1}{g}\right)^2 + \left(\frac{\theta}{2\pi}\right)^2}$$

$$\bar{n}^j \to \bar{m}^j \quad ; \quad \bar{m}^j \to -\bar{n}^j$$

$$\{\nu^j\} \to \{\mu^j\} \quad ; \quad \{\mu^j\} \to -\{\nu^j\} \tag{19}$$

(Notice that, in order to define symmetries of our model, we have to make the transformations to act on the charge densities as well as on the background.)

The above discrete transformations are more easily described in terms of the complex parameter:

$$\zeta \equiv \frac{\theta}{2\pi} + i\frac{1}{g}$$

on which the transformations $\hat{S}$, $\hat{T}$ act as follows:

$$\hat{T} \quad : \quad \zeta \to \zeta + 1$$

$$\hat{S} \quad : \quad \zeta \to -\frac{1}{\zeta} \tag{20}$$

Then $\hat{S}$ and $\hat{T}$ generate the group $\mathcal{G}$ which acts on ζ as:

$$\mathcal{G} \quad : \quad \zeta \to \frac{A\zeta + B}{C\zeta + D} \tag{21}$$

where $A, B, C, D, \in Z$ and $AD - BC = 1$, defining the discrete group $SL(2, Z)$, relevant for the analysis of the Hall plateaux hierarchy [6, 11].

In the Appendix some properties of the $SL(2, Z)$ fixed points are discussed.

3 Renormalization Group analysis.

3.1 Linear approximation in the fugacities.

We are now ready to derive the RG equations in the linear (in the fugacities) approximation and in the presence of a non-trivial background term.

Since the RG analysis is performed by rescaling the cut-off a, the Bohm-Aharonov term will be left unchanged and correspondingly the theory gets factorized with respect to the internal indices 3, 4. So for the moment we drop the internal indices and perform the linear RG analysis using the simplified version of the action:

$$S = \frac{1}{2}\sum_{i\neq k=1}^{N}\left[\frac{\mu_i\mu_k}{g} + g\left(\nu_i + \frac{\theta}{2\pi}\mu_i\right)\left(\nu_k + \frac{\theta}{2\pi}\mu_k\right)\right]\ln\left|\frac{\vec{r}_i - \vec{r}_k}{a}\right| +$$

$$\frac{1}{4}\sum_{i=1}^{N}\left[\frac{\bar{M}\mu_i}{gA} + \frac{g}{A}\left(\nu_i + \frac{\theta}{2\pi}\mu_i\right)\left(\bar{N} + \frac{\theta}{2\pi}\bar{M}\right)\right] r_i^2 \tag{22}$$

By rescaling a as: $a \to a(1+\lambda)$ ($\lambda = \frac{da}{a}$) the action S gets transformed as follows:

$$S \to S - \frac{\lambda}{2}\sum_{i\neq j=1}^{N}\left[\frac{\mu_i\mu_j}{g} + g\left(\nu_i + \frac{\theta}{2\pi}\mu_i\right)\left(\nu_j + \frac{\theta}{2\pi}\mu_j\right)\right] =$$

$$S+\frac{\lambda}{2}\sum_{j=1}^{N}\left[\frac{(\bar{M}+\mu_j)\mu_j}{g}+g\left(\bar{N}+\nu_j+\frac{\theta}{2\pi}(\bar{M}+\mu_j)\right)\left(\nu_j+\frac{\theta}{2\pi}\mu_j\right)\right] \tag{23}$$

where the neutrality conditions (see eq.(17)) have been taken into account.

Being the integration measure for each particle $\frac{d^2r_j}{a^2}$, we find that the change in the Partition Function may be reabsorbed in terms of the following correction to the fugacities:

$$Y(\nu,\mu)\rightarrow Y(\nu,\mu)+dY(\nu,\mu) \tag{24}$$

where:

$$dY(\nu,\mu)=\lambda\left[2+\frac{1}{2}\left(\frac{(\bar{M}+\mu)\mu}{g}+g\left(\bar{N}+\nu+\frac{\theta}{2\pi}(\bar{M}+\mu)\right)\left(\nu+\frac{\theta}{2\pi}\mu\right)\right)\right]Y(\nu,\mu) \tag{25}$$

The scaling index for the fugacity is then given by:

$$x(\nu,\mu)=2+\frac{(\bar{M}+\mu)\mu}{2g}+\frac{g}{2}\left(\bar{N}+\nu+\frac{\theta}{2\pi}(\bar{M}+\mu)\right)\left(\nu+\frac{\theta}{2\pi}\mu\right) \tag{26}$$

We shall now assume that the charge which condense will correspond to the maximum possible value of the scaling indices, for a fixed background and particle number.

That is, by taking into account the constraints expressed by eq.(17), we have to maximize

$$\xi(\nu,\mu)=x(\nu,\mu)+\alpha(\sum_{j=1}^{N}\nu_j+\bar{N})+\beta(\sum_{j=1}^{N}\mu_j+\bar{M})$$

where α and β are Lagrange multipliers.

We find:

$$\alpha=-\frac{N-2}{N}g\left[\bar{N}+\frac{\theta}{2\pi}\bar{M}\right]$$

$$\beta=-\frac{N-2}{N}\left[\bar{M}\left(\frac{1}{g}+g\left(\frac{\theta}{2\pi}\right)^2\right)+g\frac{\theta}{2\pi}\bar{N}\right] \tag{27}$$

and the maximum is reached for the following values of the "condensing" dyons:

$$\nu_1=\ldots=\nu_N=-\frac{\bar{N}}{N}\equiv\nu$$

$$\mu_1=\ldots=\mu_N=-\frac{\bar{M}}{N}\equiv\mu \tag{28}$$

From the above equations we see that the condensate is made out of N dyons whose charges are $1/N$ times the corresponding backgrounds. In particular for filling $f = \frac{1}{m}$, we have:

$$f \equiv \frac{\#\text{ of electrons}}{\#\text{ of available states}} = \frac{\bar{N}}{\bar{M}}\frac{hc}{e^2} = \frac{\bar{N}}{\bar{M}}$$

(being $h = c = e = 1$ in our units) and by using eq.(28) we get:

$$\nu = 1 \quad ; \quad \mu = m$$

Although the result in eq.(28) looks independent of the initial values of the parameters g and θ, the latter are important for the stability of the plateaux, as it will be seen in the next section in the context of the non-linear approximation of the RG equations.

We then see that the most relevant consequence of the introduction of a background is the presence of condensate phases where the charges are all equal and with the same sign, embedded in an uniform neutralizing background. Such phases are simply described in terms of the Coulomb Gas action:

$$S_N = \left[\frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right]\sum_{i\neq k=1}^{N} \ln\left|\frac{\vec{r}_i - \vec{r}_k}{a}\right|$$

$$-\frac{1}{2}\left[\frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right]\left[\frac{\bar{m}^2}{g} + g\left(\bar{n} + \frac{\theta}{2\pi}\bar{m}\right)^2\right]\sum_{k=1}^{N}|\vec{r}_k|^2 \tag{29}$$

which is a generalization of Laughlin's plasma description. For a more detailed discussion about this point see [7].

3.2 Non linear terms.

We can now work on the consistency of our model by studying the flow of the parameters g and θ, induced by non-linear corrections to the RG equations.

Our technique is a simplified version of the one used in [10]. The basic idea is to begin with a configuration of the system, described by the action S_N, in which N equal charges have condensed and to perturb the system around such a configuration by creating a pair of charges equal in modulus but with opposite signs.

Let $(\nu,\mu)$and $(-\nu,-\mu)$ be the charges created with fugacities $Y(\nu,\mu)$ and $Y(-\nu,-\mu)$ and let Y_P^2 be the fugacity corresponding to the particle-antiparticle pair:

$$Y_P^2 = Y(\nu,\mu)Y(-\nu,-\mu) \tag{30}$$

When we vary the scale a, Y_P^2 scales with an exponent given by:

$$x_{Y_P^2} = x(\nu,\mu) + x(-\nu,-\mu) = 2\left[2 - \frac{\mu^2}{2g} - \frac{g}{2}\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right]$$

From the above equation it is evident that the values of the "bare" parameters (i.e., of the parameters at the scale at which we started) for which the process of creation of a pair is relevant must satisfy the condition:

$$2 - \frac{\mu^2}{2g} - \frac{g}{2}\left(\nu + \frac{\theta}{2\pi}\mu\right)^2 > 0 \tag{31}$$

This defines a region in the plane of the parameters $(\frac{1}{g}, \frac{\theta}{2\pi})$ (see figure), which exactly coincides with the one found in [8, 9] for the case in which there is no background at all (we refer to [8, 9] for a detailed discussion about the shape of the phase diagram), but the original model is characterized by phases where the condensates of charges have both signs, i.e., they are trivial from a RG point of view. For a Hall system the condensate is made of charges with the same sign whose total charge is neutralized by an uniform background, i.e., the model is chiral. Our model allows for a description consistent with the physics of the Hall condensates [1], as it will be shortly shown. The relevant observation is that the phase diagram derived in [8, 9] continues to be valid in our model, but only if applied to (neutral) excitations around the Hall condensate.

In the action for the $N+2$ particles, S_{N+2}, the scale a is sent into $a + da$. At this point we have to sum over the configurations in which the centers of the charges in the pair are at a distance between a and $a + da$ ("particle fusion") The final result gives an action which is the same as the one for N particles but with the parameters g and θ renormalized. In this way we derive the RG equations for the parameters.

The action for $N+2$ particles is:

$$S_{N+2} = \left[\frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right] \{ \sum_{i\neq j=1}^{N} \ln\left|\frac{\vec{r}_i - \vec{r}_j}{a}\right| + \sum_{j=1}^{N} \ln\left|\frac{\vec{r}_j - \vec{r}_+}{a}\right|$$

$$- \sum_{j=1}^{N} \ln\left|\frac{\vec{r}_j - \vec{r}_-}{a}\right| \} - \left[\frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right] \ln\left|\frac{\vec{r}_+ - \vec{r}_-}{a}\right|$$

$$- \frac{1}{2}\left[\frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2\right]\left[\frac{\bar{m}^2}{g} + g\left(\bar{n} + \frac{\theta}{2\pi}\bar{m}\right)^2\right]\left[\sum_{j=1}^{N}\frac{|\vec{r}_j|^2}{A} + \frac{|\vec{r}_+|^2}{A} - \frac{|\vec{r}_-|^2}{A}\right] \tag{32}$$

where $\vec{r}_+$ and $\vec{r}_-$ denote the location of the positive and negative charge respectively.

We rescale a into $a + da$, with $da = \lambda a$ and $\lambda \ll 1$. Let us define $\vec{s} = \vec{r}_+ - \vec{r}_-$ and $\vec{R} = (\vec{r}_+ + \vec{r}_-)/2$ and sum over the configurations with $a < |\vec{s}| < a + da$.

The partial sum in the partition function will be given by:

$$Y_P^2 \int_{|\vec{s}|=a}^{|\vec{s}|=a+da} \frac{d^2 s}{a^2} \int \frac{d^2 R}{a^2} e^{S_{N+2}} \tag{33}$$

where S_{N+2} is understood to be expressed in terms of the new variables $\vec{s}$ and $\vec{R}$.

By expanding to second order in $\vec{s}/a$ we get:

$$\int \frac{d^2 s}{a^2} \left|\frac{\vec{s}}{a}\right|^{\alpha_q^2} e^{S_N} \left\{ 1 + \frac{(\alpha_q^2)^2}{2} s^a s^b \frac{\partial\psi(\vec{R})}{\partial R^a} \frac{\partial\psi(\vec{R})}{\partial R^b} \right\}$$

where:

$$\alpha_q^2 = \frac{\mu^2}{g} + g\left(\nu + \frac{\theta}{2\pi}\mu\right)^2$$

and:

$$\psi(\vec{R}) = \sum_{j=1}^{N} \ln\left|\frac{\vec{r}_j - \vec{R}}{a}\right| - \frac{\vec{R}^2}{2A}$$

We finally get, after integrating:

$$e^{S_N} + \int \frac{d^2 s}{a^2}\frac{d^2 R}{a^2} Y_P^2 e^{S_{N+2}} \approx$$

$$e^{S_N}\left\{ 1 - Y_P^2 \frac{\pi\lambda}{4}(\alpha_q^2)^2 \left[\sum_{i\neq j=1}^{N} \ln\left|\frac{\vec{r}_i - \vec{r}_j}{a}\right| - \sum_{j=1}^{N} \frac{|\vec{r}_j|^2}{2A} \right] \right\} \tag{34}$$

As a consequence α_q gets renormalized according to:

$$\alpha_q^2 \to \alpha_q^2 + \delta\alpha_q^2$$

where

$$\delta\alpha_q^2 = -Y_P^2 \frac{\pi\lambda}{4}(\alpha_q^2)^2 \tag{35}$$

Defining the scale parameter ρ through the relation:

$$d\rho = \frac{\pi\lambda}{4} Y_P^2 = \frac{\pi}{4} Y_P^2 \frac{da}{a} \tag{36}$$

we find the following differential equation:

$$\frac{d\alpha_q^2}{d\rho} = -(\alpha_q^2)^2 \tag{37}$$

with the solution given by:

$$\alpha_q^2(\rho) = \frac{\alpha_q^2(\rho_0)}{1 + \alpha_q^2(\rho_0)(\rho - \rho_0)} \tag{38}$$

from which we obtain:

$$\lim_{\rho\to\infty} \alpha_q^2(\rho) = 0 \tag{39}$$

Being α_q^2 a positive-definite quadratic form, the above equation implies for the parameters $\frac{1}{g}$ and $\frac{\theta}{2\pi}$.

$$\lim_{\rho\to\infty} \frac{1}{g(\rho)} = 0$$

$$\lim_{\rho\to\infty} \frac{\theta(\rho)}{2\pi} = -\frac{\nu}{\mu} \tag{40}$$

Notice that the modular covariance of the RG results so obtained simply relies on the fact that α_q^2 is a modular invariant object.

3.3 Comments about the solutions of the Renormalization Group equations in the non-linear approximation

The results expressed in eq.(40) not only are in agreement with the picture we presented in section 3.1 according to which the most probable vacuum condensate in our model describes the Laughlin plasma at the plateaux for a Quantum Hall fluid, but gives also a consistent physical interpretation of the parameters $\frac{1}{g}$ and $\frac{\theta}{2\pi}$ appearing in the model. In fact

$$\lim_{\rho\to\infty} \frac{1}{g(\rho)} = 0$$

corresponds to the absence of longitudinal conductance in a Hall system in the thermodynamic limit and $\frac{1}{g(\rho)}$ is naturally interpreted as the bare longitudinal conductance, σ_L.

On the other hand from eq.(40) the parameter $\frac{\theta(\rho)}{2\pi}$ can be interpreted as the bare transverse (Hall) conductance. Infact if in the thermodynamic limit the condensate is made out of dyons with electric charge ν and magnetic charge μ, the transverse conductance σ_H will be equal to the ratio between the two charges, as it must be for a Hall system [7]. Then the I.R. fixed points of $SL(2,Z)$ in which certain phases condense are interpreted as points corresponding to a Hall system at different fillings, whose transverse conductance is given by the ratio $\frac{\nu}{\mu}$.

Also let us remind that in a real system the values of the physical parameters are understood to be evaluated at a scale corresponding to the size of the system. For a finite system, if the size is smaller than the localization length ξ, the bare Hall conductance (i.e., the conductance evaluated at the size of the system), will be proportional to the reciprocal of the external magnetic field B. This means that a change in the external field is equivalent in our model to a horizontal variation (at constant $g(\rho)$) of the bare parameter $\frac{\theta(\rho)}{2\pi}$ [12, 13]. Such an observation gives us an immediate interpretation of the phase boundaries as “boundaries” of the plateaux which at fixed disorder has a size given by the horizontal distance between the phase boundaries enclosing it (see figure).

Obviously the experimental results have to be compared with the values of the parameters in the thermodynamic limit, which does not depend on the initial point, if the phase does not change. But it is also important to give a correct interpretation of the phase boundaries as the boundaries between plateaux. Such lines are not, strictly speaking, marginal lines, but they are attracted by the RG fixed point in the middle, which is repulsive in every direction except along the phase boundaries. These points should be identified with the middle points of the slopes between plateaux. In this framework the presence of the $SL(2,Z)$ symmetry is at the basis of the universality of the critical indices of the transition [7] in agreement with the experimental results. Furthermore the unstable fixed points seem to be described by a 2D CFT with central charge $c=0$ [14, 15], which is the field theoretical description of the percolative fixed point [16], and presently we are working on this point (see also section 5).

4 The repulsive RG fixed points: a proposal.

4.1 General description of the repulsive fixed point.

In this section we study what happens at the transition between two distinct phases. This is done in order to understand the behavior of the system close to the saddle-point and to show how we can map it onto a percolative model, according to the analysis given in [16]. Thanks to the modular symmetry $SL(2,Z)$ it is sufficient to study the behavior of the model around only one saddle-point in order to derive the values of the physical quantities at all the other points, in agreement with recent experimental results supporting universality of the critical properties at the transition between plateaux [17].

Let us give a picture of the transition between the two condensate phases

$$(\bar{\nu},\bar{\mu})=(1,-1) \quad \text{and} \quad (\bar{\nu},\bar{\mu})=(2,-1)$$

from now on to be referred to as (1) and (2) respectively.

The transition saddle-point that separates them is given by:

$$\left(\frac{1}{g^*},\frac{\theta^*}{2\pi}\right)=\left(\frac{1}{2},\frac{3}{2}\right)$$

and we will suppose that, once we fix the values of the parameters at the critical point, the system will be constituted by a "mixture" of an equal number of (1) and (2) - particles, let us say N type-(1) particles and N type-(2) ones.

Starting from such a hypothesis we will construct a mapping of our model to a percolative transition model. This turns out to be very useful a lattice version of the model defined by the action:

$$S_0[\vec{r}^{(1)}_1,\ldots,\vec{r}^{(1)}_N,\ \vec{r}^{(2)}_1,\ldots,\vec{r}^{(2)}_N]=$$

$$\frac{g^*}{2}\sum_{\vec{r}^{(1)}_i\neq\vec{r}^{(1)}_j}(1-\frac{\theta^*}{2\pi})^2G(\vec{r}^{(1)}_i-\vec{r}^{(1)}_j)+\frac{1}{2g^*}G(\vec{r}^{(1)}_i-\vec{r}^{(1)}_j)+\frac{g^*}{2}\sum_{\vec{r}^{(2)}_i\neq\vec{r}^{(2)}_j}(2-\frac{\theta^*}{2\pi})^2G(\vec{r}^{(2)}_i-\vec{r}^{(2)}_j)+G(\vec{r}^{(2)}_i-\vec{r}^{(2)}_j)+$$

$$g^* \sum_{\vec{r}_i^{(1)},\vec{r}_j^{(2)}} (1-\frac{\theta^*}{2\pi})(2-\frac{\theta^*}{2\pi}) G(\vec{r}_i^{(1)}-\vec{r}_j^{(2)}) + \frac{1}{g^*} \sum_{\vec{r}_i^{(1)},\vec{r}_j^{(2)}} G(\vec{r}_i^{(1)}-\vec{r}_j^{(2)}) +$$

$$i \sum_{\vec{r}_i^{(1)},\vec{r}_j^{(2)}} \varphi(\vec{r}_i^{(1)}-\vec{r}_j^{(2)}) +$$

$$\sum_{\vec{r}_j^{(1)}} \left[\frac{g^*}{4}(1-\frac{\theta^*}{2\pi})^2 + \frac{1}{4g^*} \right] (\vec{r}_j^{(1)})^2 + \sum_{\vec{r}_j^{(2)}} \left[\frac{g^*}{4}(2-\frac{\theta^*}{2\pi})^2 + \frac{1}{4g^*} \right] (\vec{r}_j^{(2)})^2 \tag{41}$$

where $\vec{r}_1^{(1)}, \ldots, \vec{r}_N^{(1)}$, $\vec{r}_1^{(2)}, \ldots, \vec{r}_N^{(2)}$ define the locations of the two types of particles and $G(\vec{r}_i - \vec{r}_j)$ and $\varphi(\vec{r}_i - \vec{r}_j)$ are the Green functions on the lattice.

We now need to make a second basic hypothesis, i.e., that S_0 does not depend on the locations of the particles. Such a statement could be easily proven on a torus, but for the moment we shall leave it as a guess.

Next step will be to study what happens when we "move" a little away from the critical point. In order to do so, let us introduce the fugacities $Y(1)$ and $Y(2)$ for type-(1) and type-(2) particles respectively. At the fixed point we have $Y(1) = Y(2)$ and the system may be represented as a lattice with equal number of particles of both types on its sites.

To move away from the critical point let us suppose that $Y(1) > Y(2)$. In particular we shall assume that we are changing the value of the parameter $\frac{\theta}{2\pi}$ from its critical value, $\frac{\theta^*}{2\pi}$ to:

$$\frac{\theta}{2\pi} = \frac{\theta^*}{2\pi} + \frac{\delta\theta}{2\pi}$$

Once we made such a change there will be two possible configurations:

- The state to which we shall refer to as the "ground state" , $|0\rangle$, which is a superposition of states with equal number of particles of the two types;
- The state that has $N+1$ type-(1) particles and $N-1$ type-(2) particles obtained after changing the type of particle lying on a specific site from (2) to (1).

Let us now focus our attention on a given site of the lattice, say 1.

We will suppose that at the critical point site 1 is occupied by a type-(2) particle. When we move away from the critical point, we can evaluate the relative probability that 1 is still occupied by a type-(2) particle or that it is occupied by a type-(1) particle. We find that the weight corresponding to the configuration with site 1 occupied by a type-(1) particle is:

$$w_1 = k[Y(1)Y(2)]^N \frac{Y(1)}{Y(2)} \exp\{-S_1\}$$

where S_1 is the action corresponding to a configuration in which the sites $\vec{r}_1^{(1)}, \ldots, \vec{r}_N^{(1)}, \vec{r}_1^{(2)}$ are occupied by a type-(1) particle while the sites $\vec{r}_1^{(2)}, \ldots, \vec{r}_N^{(2)}$ are occupied by a type-(2) one. The combinatorial factor k takes into account permutations among identical particles and is equal to:

$$k = \frac{(2N-1)!}{N!(N-1)!}$$

The previous two equations are justified if for the action S_1 we make the same hypothesis as the one made for the action S_0 (and we shall assume that it may be proven in the same framework), i.e., that it <u>does not depend on the locations of the particles</u>.

The weight corresponding to a configuration in which site 1 is occupied by a type-(2) particle will be:

$$w_2 = k(Y(1)Y(2))^N e^{-S_0}$$

Let us finally write down the probabilities p_1 for a type-(1) particle on (1) ("filled site") and p_0 for a type-(2) particle ("empty site"):

$$p_1 = \frac{w_1}{w_1 + w_2} \approx \frac{Y(1)}{Y(1) + Y(2)}$$

$$p_0 \approx \frac{Y(2)}{Y(1) + Y(2)} \tag{42}$$

Again we made an assumption about the relative values of the action, i.e. that:

$$e^{S_1^* - S_0^*} \approx 1 \tag{43}$$

where the $*$ symbol indicates that the parameters are to be evaluated at the critical point.

Let us now sketch how our model can be mapped onto a percolative model.

4.2 Mapping onto a percolative model.

The "percolative model" we shall look at is defined as follows:

- A 2-dimensional lattice with $2N$ sites each of which may be "occupied" by a particle or "empty";
- A probability p for the site being occupied that is site-independent.

It is well-known that such a model presents a phase transition when p increases to its critical value $p_c = .5$. Such a transition appears when islands of filled sites become a connected cluster that "percolates" throughout the lattice. The physical quantity that characterizes such a transition is the "correlation length" ξ that measures the mean extension of a big cluster. When p approaches p_c, ξ diverges with a power-law as:

$$\xi \sim |p - p_c|^{-\nu} \tag{44}$$

where the exponent ν is equal to $\frac{4}{3}$.

The correspondence with our model gets traced once we define the probability p of the percolative model as:

$$p = p_1 = \frac{Y(1)}{Y(1) + Y(2)}$$

This is in agreement with the fact that at the critical point we must have $p = .5$, corresponding to $Y(1) = Y(2)$.

Since we are able to perform a RG analysis in our model by scaling the cutoff length a to $(1+\lambda)a$ we can relate the index ν to the scaling exponents of the parameters of our model. In particular we shall refer to the scaling properties (at the transition between Hall plateaux) described in [13] in terms of $\delta\sigma_H$. It has been proven there that, if we rescale length L by b, close to the critical point the quantity $\delta\sigma_H = \sigma_H - \sigma_H^*$ scales according to:

$$\delta\sigma_H \sim b^{-\frac{1}{\nu}} \tag{45}$$

On the other hand by looking at the mapping introduced above we find:

$$|p - p_c| \sim |Y(1) - Y^*(1)|$$

Furthermore we shall assume that $Y(1)$ deviates from its critical value $Y^*(1)$ due to a deviation in the parameter $\frac{\theta}{2\pi}$ from its critical value $\frac{\theta^*}{2\pi}$. Being $Y(1)$ a smooth function of $\frac{\theta}{2\pi}$ (as it may be easily seen from the scaling equations given above), we can write down:

$$Y(1) - Y^*(1) \approx \frac{\partial Y^*(1)}{\partial \frac{\theta}{2\pi}} \frac{\delta\theta}{2\pi} \tag{46}$$

where $\delta\theta = \theta - \theta^*$ and

$$\frac{\partial Y^*(1)}{\partial \frac{\theta}{2\pi}}$$

is finite.

Finally from eqs.(44, 46) we get:

$$\xi(\theta, \sigma) \sim \left| \frac{\delta\theta}{2\pi} \right|^{-\nu}$$

that is:

$$\delta\left(\frac{\theta}{2\pi}\right) \sim \xi^{-\frac{1}{\nu}} \tag{47}$$

The above equation reproduces the scaling properties described in eq.(45) once we identify $\delta\left(\frac{\theta}{2\pi}\right)$ as $\sigma_H - \sigma_H^*$, in agreement with our RG analysis given in section 3.

5 Summary, comments and suggestions.

In this paper we analyzed in detail the long-distance properties of a dual model recently proposed for describing a Quantum Hall fluid [7]. Even though duality has been lately introduced by several authors for describing universality, we feel that our proposed model enjoies two essential features:

1. It is general. It allows for dyon condensation in the vacuum by introducing (a la ′t Hooft) a coupling $\frac{\theta}{2\pi}$ between the electric and magnetic charges. Consequently the duality enjoied by the model simply describes not only the usual symmetry under the exchange of the electric and magnetic charges but also the symmetry under electric (magnetic) charge translation by integer values.

2. The physical interpretation of the two parameters appearing in the model is not guessed but comes out from a detailed (non-linear) RG analysis describing their flow.

By fixing the values of the parameters at the transition between two phases, with reasonable hypothesis we traced a mapping between our description and the classical percolation model. By using the scaling properties of the $\frac{\theta}{2\pi}$ parameter we were led to fix the critical exponent ν to the value $\nu = \frac{4}{3}$.

We would like to give now some suggestions.

Even though there is no clear experimental result for ν there is a general consensus for its value to be $\nu = \frac{7}{3}$ [18].

Can quantum tunneling provide an enhancement of the percolation between the two phases in such a way that:

$$\nu = \frac{4}{3} \to \nu = \frac{7}{3} \ \ ?$$

How to describe quantum tunneling in our model?

Further is it possible to describe the critical properties of the system in terms of a 2D CFT? What would be the value of its central charge?. The percolative picture just presented would suggest a 2D CFT with central charge $c = 0$ as analytic extension of the unitary series described by a theory with $c = 1 - \frac{6}{m(m+1)}$.

In fact it is possible to perturb around the critical point by employing thermal operators, with an Effective Action given by:

$$S^* + \Delta S \tag{48}$$

where S^* describes the system at criticality and the "perturbation" ΔS is given by [14]:

$$\Delta S = \Delta \left(\frac{\theta}{2\pi}\right) \int d^2 z \Phi_{13}(z, z^*) \tag{49}$$

(Φ_{13} is a thermal operator with conformal dimension $h = \frac{5}{8}$ [15]).

It is straightforward to show that the relation between ν and h is given by:

$$\nu = \frac{1}{2(1-h)} \tag{50}$$

obtaining for the critical exponent ν the value $\nu = \frac{4}{3}$.

An obvious comment is that the above argument is just naive. We think that it would be interesting to define with care the limit $m \to 2$ in order to obtain a correct interpretation of the $c = 0$ theory and then to be able to evaluate the critical index.

At the moment we are workng on this point.

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Appendix: $SL(2,Z)$ fixed points and their relation with the Renormalization Group.

A fixed point of $SL(2,Z)$ is a complex number z_* such that it exists al least one $A \in SL(2,Z)$ such that $Az_* = z_*$ and $A \neq 1$.

We can easily prove the following theorem:

"If z_* is a fixed point of $SL(2,Z)$ then $\forall B \in SL(2,Z)$ Bz_* is a fixed point too".

To prove such a statement we begin by reminding that there is $A \in SL(2,Z)$ such that $Az^* = z^*$. If we define $C = BAB^{-1}$, then we get:

$$C(Bz^*) = Bz^*$$

which completes the proof.

It is not difficult to prove that the classification of the fixed points is as follows:

- $i\infty$, invariant under $\hat{T}$;
- i, invariant under $\hat{S}$;
- $\frac{1}{2} + i\frac{\sqrt{3}}{2}$, invariant under $\hat{T}\hat{S}$.

Finally let us notice that, by applying $SL(2,Z)$ to $i\infty$ we generate the set of rational numbers:

$$SL(2,Z)(i\infty) = Q = \{\frac{p}{q}\}, q \neq 0$$

An important property of the fixed $SL(2,Z)$ points is that they are also fixed RG points (although the vice versa is not necessarily true). For simplicity we shall assume that the set of RG fixed points is "minimal", i.e., that the fixed RG points exactly coincide with the fixed points of $SL(2,Z)$.

Since $\hat{S}^2 = 1$ e $(\hat{T}\hat{S})^3 = 1$, the points generated by the application of $SL(2,Z)$ to i will be double points of the RG while the ones generated by $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ will be triple ones.

The assignment of the RG flux lines is chosen in such a way that:

- it agrees with the "Physical interpretation" of the parameters;
- it agrees with the RG flux for the parameters g and θ.

Let us now emphasize the physical role of those fixed points:

- $\underline{C_1 \text{ fixed points}}$ will be the point $i\infty$ and its images under the group $SL(2,Z)$. They will be identified as attractive RG fixed points. This means that they are the points attracting the condensate phases of the system in the large scale limit. Then all the rational numbers are I.R. fixed points. This is a relevant consequence of the generalized duality symmetry and is at the basis of a correct description of the hierarchy.

- C_2 fixed points They separate two phases corresponding to two different Hall condensates. The RG flux lines assignment shows that these points have the topology of "saddle points", i.e., they are attractive in every direction but along the phase boundary. Then it is evident that they should be identified with the transition points between plateaux and so they are localization/delocalization transition points.

- C_3 fixed points We will not spend much time in dealing with them since they will disappear when the modular group $SL(2, Z)$ gets restricted to $\Gamma(2)$ in order to match the actual physical problem, i.e. the odd denominator rule [15]. What we could tell about them is that they correspond to the opening of new phases and they are repulsive in each direction.

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Figure caption:

It is shown the phase diagram of the model in the plane $\left(\frac{1}{g}, \frac{\theta}{2\pi}\right)$.

The attractive IR fixed points are indicated with the symbol $\oplus$, the saddle points with $\otimes$ and the triple points with $\ominus$.